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On nonclassical boundary conditions for the contact of thin interlayers with different physical and mechanical properties on wave propagation in anisotropic media

Wave processes are intensively studied in various fields of physics: electrodynamics, plasma physics, radiophysics, acoustics, hydrodynamics, etc. Along with the study of electromagnetic and elastic wave processes, the research of patterns of wave propagation of various physical nature in the presence of mutual transformation are of particular relevance. Wave processes in coupled fields reflect the mutual influence of elastic, electromagnetic and thermal fields. The coupling of electromagnetic fields to the deformation field takes place in a medium with piezoelectric, piezomagnetic and magnetostrictive properties. In the paper, based on the matrix method, the propagation of coupled elastic and electromagnetic waves in media with different physical and mechanical properties is studied. The paper proposes a generalization of non-classical contact conditions for studying the effect of thin layers with different physical and mechanical properties on wave processes. A system of differential equations of the 1st order with variable coefficients is constructed, which describe the propagation of electroelastic waves in anisotropic media of a rhombic system of class 222. The conditions for nonrigid contact for a thin layer with piezoelectric properties are derived. The possibility of studying layers with δ-shaped properties (δ is the Dirac function) is proved.

Keywords: Maxwell's equations, anisotropic medium, waves, non-rigid contact, matricant method.

Introduction

Scientific interest in interrelated elastic and electromagnetic wave processes in media with piezoelectric, piezomagnetic, and thermopiezoelectric properties has recently been associated with the prospect of application in various fields of science and technology, such as instrumentation, micro and nanoelectronics, and information technology. It is possible to allocate applications in high-frequency electronics; the use of multiferroic structures in various kinds of logical elements, memory elements and information processing devices; autonomous wireless energy sources; sensors of variable and constant fields; creation of new composite materials [1–4].

In [5–11], a theoretical model was proposed that describes the properties of composite layered composites based on magnetostrictive and piezoelectric materials in the low-frequency range and the region of electromechanical resonance. The theory is presented and the magnetoelectric effect in multilayer composite materials based on ferrite-piezoelectrics for samples of various shapes and in a two-layer magnetostrictive-piezoelectric structure is experimentally investigated. Theoretical research is conducted not only by Russian scientists, but also by representatives of other countries. For example, in the article [12], the magnetoelectric effect is studied in magnetostrictive layers. Various experimental methods for measuring the magnetoelectric effect are considered in [13–15] and an energy source based on magnetostrictive piezoelectric composites has been designed.

Along with the study of electromagnetic and elastic (acoustic) wave processes, studies of the patterns of propagation of waves of various physical natures in the presence of mutual transformation are of particular relevance. Wave processes in coupled fields reflect the mutual influence of elastic, electromagnetic, and thermal fields. The connection of electromagnetic fields with the deformation field takes place in a medium with piezoelectric, piezomagnetic, and magnetostrictive properties.

Studies of new physical phenomena and the creation of devices and devices for solid-state electronics are largely associated with the synthesis of materials with new specified properties, the manufacture of multifunctional composite materials consisting of two or more separate phases [16–20].

Research method

Based on the matricant method [21], we study the propagation of coupled elastic and electromagnetic waves in media with different physical and mechanical properties and the use of these features for practical purposes; as well as the development of a method for determining the averaged physical and mechanical parameters of heterostructures in the presence of coupled fields.

Wave processes in elastic anisotropic media, in anisotropic dielectric media, waves in anisotropic plates, electromagnetic waves in media with a magnetoelectric effect [22–26], waves in liquid crystals [27], wave propagation in thermoelastic media [28–30], related wave processes in media with piezoelectric and piezomagnetic effects [31]. On its basis, a unified description of Rayleigh-type surface waves, Lamb-type waves in elastic, piezoelectric, piezomagnetic media and media with a magnetoelectric effect was obtained [32–34].

Basic Equations and Relations

In this paper, we discuss the possibility of applying the matricant method to the study of wave propagation in multilayer heterostructures.

In the presence of layers that satisfy the condition $\lambda >> l_i$ (λ is the wavelength, l_i is the thickness of the i-th layer), the construction of wave field solutions and their analysis can be significantly simplified. In this case, the influence of thin layers is considered by special (non-classical) boundary conditions.

Considering the influence of thin layers by means of boundary conditions makes it possible to exclude the construction of solutions to the equations of motion in these layers, which naturally significantly reduces the number of calculations and facilitates the analysis of the obtained solutions.

For the first time, the boundary conditions of a non-rigid contact were proposed in the work of Podyapolsky G.S. [35]. The main purpose of their introduction was to consider the contact conditions occupying an intermediate position between hard contact:

$$\vec{W}_i \Big|_{z=0} = \vec{W}_z \Big|_{z=0} \qquad (1)$$

and free surface based on the introduction of thin viscoelastic layers. The substantiation of the introduction of nonrigid contact conditions, as well as the derivation of these conditions, had a number of limitations. Disadvantages were discussed in [36]. In this work, the application of the boundary conditions of non-rigid contact received a deeper physical justification and content. In addition, in the same article, a general algorithm for deriving the boundary conditions of a nonrigid contact was proposed, considering the rheological properties of the interlayer and inertial effects following from the corresponding equations of motion. A wide class of models of continuous media and motion was considered. A wide class of models of continuous media was considered and the boundary conditions of nonrigid contact were obtained, which describe the boundary conditions of nonrigid contact, which describe the influence of the corresponding thin layers on wave processes.

A generalization of the original version of the boundary conditions of a nonrigid contact was carried out in [37]. Some applied issues of hard contact are given in [37]. Boundary conditions for non-rigid contact are considered in the monograph by L.A. Molotkov [38].

In the works noted above and in other scientific publications, the application of the boundary conditions of nonrigid contact was limited to various models of solid mechanics media. At the same time, the area of constructive application of the boundary conditions of nonrigid contact is very extensive.

The derivation of boundary non-rigid contact for elastic anisotropic media with various physical and mechanical phenomena is simple and understandable on the basis of the matricant method. In the case of a piezoelectric elastic medium, the equations of motion:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial^2 U}{\partial x^2} \qquad (2)$$

and Maxwell's equations in differential form:

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$rot\vec{H} = -\frac{\partial \vec{D}}{\partial t} \qquad (3)$$
$$div\vec{B} = 0$$
$$div\vec{D} = 0$$

connected by defining relations:

$$\sigma_{ij} = e_{ijkl}\varepsilon_{kl} - e_{kil}E_k$$
$$D_i = e_{ijkl}\varepsilon_{kl} + э_{ik}E_k \qquad (4)$$
$$B_i = \mu_0\mu_{ij}H_j$$

where c_{ijkl} – elastic stiffness, ρ – medium density, $\varepsilon_{kl} = \frac{1}{2}(u_{l,k} + u_{k,l})$ – strain tensor, e_{ikl} – piezoelectric constants relating the electric field to mechanical stresses; $э_{ik}$ – components of the permittivity tensor.

Construction of a system of differential equations of the 1st order. Analysis of coefficient matrices

Consider a rhombic system of class 222.

The rhombic system of class 222 is a system with three mutually perpendicular axes, which are double axes of symmetry. Such a system must correspond to two class 2 monoclinic systems: one with a twofold symmetry axis parallel to the Y axis and the other with a twofold symmetry axis parallel to the Z axis. The material constants must be determined by both monoclinic systems. This condition leads to a decrease in the number of constants. The coefficient matrices for the class 222 rhombic system are [39]:

$$c_{ijkl} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}; e_{kij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{14} & 0 & 0 \\ 0 & e_{25} & 0 \\ 0 & 0 & e_{36} \end{pmatrix}; э_{ij} = \begin{pmatrix} э_{11} & 0 & 0 \\ 0 & э_{22} & 0 \\ 0 & 0 & э_{33} \end{pmatrix} \qquad (5)$$

Here we have 9 independent coefficients c_{ijkl} , 3 coefficients e_{kij}, and 3 coefficients $э_{ij}$.

In abbreviated matrix notation, relations (4) are written as follows:

$$\left.\begin{aligned} \sigma_p = T_p = c_{pq}S_q - e_{kp}E_k \\ D_i = e_{iq}S_{ql} + э_{ik}E_k \end{aligned}\right\} \qquad (6)$$

where *i, k = 1,2,3; p,q=1,2,3,4,5,6*

$$S_p = \varepsilon_{ij}, i = j$$
$$2\varepsilon_{ij} = S_p, i \neq j$$

We write the constitutive relations (4) in the matrix form:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} * \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{14} & 0 & 0 \\ 0 & e_{25} & 0 \\ 0 & 0 & e_{36} \end{bmatrix} * \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{36} \end{bmatrix} * \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} + \begin{bmatrix} э_{11} & 0 & 0 \\ 0 & э_{22} & 0 \\ 0 & 0 & э_{33} \end{bmatrix} * \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (8)$$

where

$$\begin{gathered} T_1 = \sigma_{11}; T_2 = \sigma_{22}; T_3 = \sigma_{33}; T_4 = \sigma_{23}; T_5 = \sigma_{31}; T_6 = \sigma_{12}; S_1 = \varepsilon_{11} = u_{1,1}; S_2 = \varepsilon_{22} = u_{2,2}; \\ S_3 = \varepsilon_{33} = u_{3,3}; S_4 = 2\varepsilon_{23} = u_{2,3} + u_{3,2}; S_5 = 2\varepsilon_{31} = u_{3,1} + u_{1,3}; S_6 = 2\varepsilon_{12} = u_{1,2} + u_{2,1}. \end{gathered} \quad (9)$$

From relation (7) follow the expressions for the components of the stress tensor:

$$\begin{aligned} \sigma_{xx} &= c_{11}\frac{\partial u_x}{\partial x} + c_{12}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} \\ \sigma_{yy} &= c_{12}\frac{\partial u_x}{\partial x} + c_{11}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} \\ \sigma_{zz} &= c_{13}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_y}{\partial y} + c_{33}\frac{\partial u_z}{\partial z} \\ \sigma_{yz} &= c_{44}\frac{\partial u_y}{\partial z} + c_{44}\frac{\partial u_z}{\partial y} - e_{14}E_x \\ \sigma_{xz} &= c_{44}\frac{\partial u_z}{\partial x} + c_{44}\frac{\partial u_x}{\partial z} - e_{14}E_y \\ \sigma_{xy} &= c_{66}\frac{\partial u_x}{\partial y} + c_{66}\frac{\partial u_y}{\partial x} - e_{36}E_z \end{aligned} \quad (10)$$

From relation (8) follow the components of electric induction:

$$\begin{aligned} D_x &= e_{14}\frac{\partial u_y}{\partial z} + e_{14}\frac{\partial u_z}{\partial y} + э_{11}E_x \\ D_y &= e_{14}\frac{\partial u_z}{\partial x} + e_{14}\frac{\partial u_x}{\partial z} + э_{11}E_y \\ D_z &= e_{36}\frac{\partial u_x}{\partial y} + e_{36}\frac{\partial u_y}{\partial x} + э_{33}E_z \end{aligned} \quad (11)$$

Given that

$$\frac{\partial f}{\partial z} = \frac{df}{dz}, \frac{\partial f}{\partial x} = -ik_x f, \frac{\partial f}{\partial y} = -ik_y f, \frac{\partial f}{\partial t} = -i\omega f \quad (12)$$

Fulfilling conditions (12), the expressions for the voltage components and electric induction (10) and (11) will take the form:

$$\begin{aligned}
\sigma_{xx} &= -ik_x c_{11} u_x - ik_y c_{12} u_y + c_{13}\frac{\partial u_z}{\partial z} \\
\sigma_{yy} &= -ik_x c_{12} u_x - ik_y c_{11} u_y + c_{13}\frac{\partial u_z}{\partial z} \\
\sigma_{zz} &= -ik_x c_{13} u_x - ik_y c_{13} u_y + c_{13}\frac{\partial u_z}{\partial z} \\
\sigma_{yz} &= c_{44}\frac{\partial u_y}{\partial z} - ik_y c_{44} u_z - e_{14} E_x \\
\sigma_{xz} &= -ik_x c_{44} u_z + c_{44}\frac{\partial u_x}{\partial z} - e_{14} E_y \\
\sigma_{xy} &= -ik_y c_{66} u_x - ik_x c_{66} u_y - e_{36} E_z \\
D_x &= e_{14}\frac{\partial u_y}{\partial z} - ik_y e_{14} u_z + э_{11} E_x \\
D_y &= -ik_x e_{14} u_z + e_{14}\frac{\partial u_x}{\partial z} + э_{11} E_y \\
D_z &= -ik_y e_{36} u_x - ik_x e_{36} u_y + э_{33} E_z
\end{aligned} \tag{13}$$

Since the inhomogeneity is assumed along the z axis, it is necessary to extract the derivatives with respect to z.

Equations of motion (2), considering conditions (12), we rewrite in the following form:

$$\begin{aligned}
-ik_x\sigma_{xx} - ik_y\sigma_{xy} + \frac{\partial\sigma_{xz}}{\partial z} &= -\rho\omega^2 u_x \\
-ik_x\sigma_{xy} - ik_y\sigma_{yy} + \frac{\partial\sigma_{yz}}{\partial z} &= -\rho\omega^2 u_y \\
-ik_x\sigma_{xz} - ik_y\sigma_{yz} + \frac{\partial\sigma_{zz}}{\partial z} &= -\rho\omega^2 u_z
\end{aligned} \tag{14}$$

Electromagnetic fields in anisotropic media in the absence of charges and currents are described by the first pair of Maxwell’s equations (3). These equations in component-wise form are written as follows:

$$\begin{aligned}
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -i\omega\mu_0\mu H_x \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -i\omega\mu_0\mu H_y \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -i\omega\mu_0\mu H_z \\
\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= i\omega D_x \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= i\omega D_y \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= i\omega D_z
\end{aligned} \tag{15}$$

Considering the initial provisions stated above, the representation of wave fields is considered in the form:

$$\vec{F}=\vec{F}(\omega,z)e^{i\omega t\pm ik_x x\pm ik_y y} \tag{16}$$

Using the representation of solutions for vectors $\vec{E}$ and $\vec{H}$ in the form (16), we obtain:

$$\begin{aligned}\vec{E}(\omega,\vec{r})&=\vec{E}(\omega,z)e^{i(\omega t-k_x x-k_y y)}\\ \vec{H}(\omega,\vec{r})&=\vec{H}(\omega,z)e^{i(\omega t-k_x x-k_y y)}\end{aligned} \tag{17}$$

and substituting relations (13) and (17) into (15), as well as excluding the components that are not included in the boundary conditions, we obtain a system of ten first-order differential equations relating the components of stresses , strains and electric and magnetic fields:

$$\frac{dU_z}{dz}=\frac{1}{c_{33}}\sigma_{zz}+\left(\frac{ik_x c_{13}}{c_{33}}\right)U_x+\left(\frac{ik_y c_{13}}{c_{33}}\right)U_y\,;$$

$$\frac{d\sigma_{zz}}{dz}=-\rho\omega^2 U_z+ik_x\sigma_{xz}+ik_y\sigma_{yz}\,;$$

$$\frac{dU_x}{dz}=\frac{1}{c_{55}}\sigma_{xz}+ik_x U_z+\frac{e_{25}}{c_{55}}E_y\,;$$

$$\frac{d\sigma_{xz}}{dz}=\frac{ik_x c_{13}}{c_{33}}\sigma_{zz}+(-\rho\omega^2+k_x^2c_{11}-\frac{k_x^2c_{13}^2}{c_{33}}+k_y^2c_{66}+\frac{k_y^2e_{36}^2}{э_{33}})U_x+(k_xk_yc_{12}-\frac{k_xk_yc_{13}c_{23}}{c_{33}}+k_xk_yc_{66}+$$
$$+\frac{k_xk_ye_{36}^2}{э_{33}})U_y-\frac{ik_y^2e_{36}}{\omega э_{33}}H_x+\frac{ik_xk_ye_{36}}{\omega э_{33}}H_y;$$

$$\frac{dU_y}{dz}=\frac{1}{c_{44}}\sigma_{yz}+ik_yU_z+\frac{e_{14}}{c_{44}}E_x\,;$$

$$\frac{d\sigma_{yz}}{dz}=\frac{ik_yc_{13}}{c_{33}}\sigma_{zz}+(-\rho\omega^2+k_x^2c_{66}+k_y^2c_{22}-\frac{k_{и}^2c_{23}^2}{c_{33}}+\frac{k_x^2e_{36}^2}{э_{33}})U_y+(k_xk_yc_{66}+\frac{k_xk_ye_{36}^2}{э_{33}}+k_xk_yc_{12}-$$
$$-\frac{k_xk_yc_{13}c_{23}}{c_{33}})U_x+\frac{ik_x^2e_{36}^2}{\omega э_{33}}H_y-\frac{ik_xk_ye_{36}}{\omega э_{33}}H_x;$$

$$\frac{dE_y}{dz}=\frac{ik_xk_y}{\omega э_{33}}H_y+(-\frac{ik_y^2}{\omega э_{33}}+i\omega\mu\mu_0)H_x+\frac{k_y^2e_{36}}{э_{33}}U_x+\frac{k_xk_ye_{36}}{э_{33}}U_y\,;$$

$$\frac{dE_x}{dz}=-\frac{ik_xk_y}{\omega э_{33}}H_x+(\frac{ik_x^2}{\omega э_{33}}-i\omega\mu\mu_0)H_y+\frac{k_xk_ye_{36}}{э_{33}}U_x+\frac{k_x^2e_{36}}{э_{33}}U_y\,;$$

$$\frac{dH_x}{dz}=\frac{i\omega e_{25}}{c_{55}}\sigma_{xz}+(i\omega\frac{e_{25}^2}{c_{55}}+i\omega э_{22}-\frac{ik_x^2}{\omega\mu\mu_0})E_y+\frac{ik_xk_y}{\omega\mu\mu_0}E_x\,;$$

$$\frac{dH_y}{dz}=-\frac{i\omega e_{14}}{c_{44}}\sigma_{yz}+(-i\omega\frac{e_{14}^2}{c_{44}}-i\omega э_{11}+\frac{ik_y^2}{\omega\mu\mu_0})E_x-\frac{ik_xk_y}{\omega\mu\mu_0}E_y\,.$$

This system of differential equations in matrix form has the form:

$$\frac{d\vec{W}}{dz} = B\vec{W}; \vec{W} = (U_z, \sigma_{zz}, U_x, \sigma_{xz}, U_y, \sigma_{yz}, E_y, H_x, H_y E_x) \quad (18)$$

where u_i, σ_{iz} – displacement vector and stress tensor components; E_y, H_x, H_y, E_x – components of electric and magnetic fields; k_x, k_y – respectively x and y are the components of the wave vector; $\hat{B} = \hat{B}\left[c_{ijkl}(z), e_{kij}(z), э_{ij}(z), k_x, k_y\right]$ – matrix of coefficients, the elements of this matrix contain the parameters of the medium in which the electroelastic waves propagate.

The matrix $\hat{B}$ in the case of propagation of electroelastic waves along the Z axis has the following structure:

$$\hat{B} = \begin{pmatrix} 0 & b_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{34} & b_{35} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{56} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\omega b_{35} & b_{65} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{78} & 0 & b_{710} \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i\omega b_{710} & 0 & b_{910} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b_{56} & 0 \end{pmatrix} \quad \vec{u} = \begin{pmatrix} U_z \\ \sigma_{zz} \\ U_x \\ \sigma_{xz} \\ E_y \\ H_x \\ U_y \\ \sigma_{yz} \\ H_y \\ E_x \end{pmatrix} \quad (19)$$

Based on the structure of the matrix of coefficients, it follows that in this case in the piezocrystal there is not one, but several types of waves, the interaction between which is determined by the coefficients $b_{35} = \frac{e_{25}}{c_{55}}$ and $b_{710} = \frac{e_{14}}{c_{44}}$. These coefficients reflect the relationship between the piezoelectric moduli and the elastic constants of the medium in which the waves propagate. An elastic longitudinal wave, described by the coefficients b_{12} and b_{21} propagates independently of other types of waves. The coefficient b_{35} determines the interaction between the elastic transverse x-polarization wave and the electromagnetic TE-wave, and the coefficient b_{710} determines the interaction between the elastic transverse wave of the y-polarization and the electromagnetic TM-wave.

The coefficients that determine the relationship between different types of waves provide a constant transition of the energy of elastic waves into the energy of electromagnetic waves and vice versa.

Boundary conditions for non-rigid contact

If in an elastic dielectric medium there is a thin layer of thickness h and $\lambda >> h$ (λ is the wavelength, h is the inhomogeneity period), then the system of equations (18) can be represented in the finite difference form:

$$\frac{d\vec{W}}{dz} \cong \frac{\Delta\vec{W}}{h} = B\vec{W}; \Delta\vec{W} = \vec{W}_2 - \vec{W}_1 \quad (20)$$

From (20) it immediately follows:

$$\vec{W}_2 = [E + Bh]\vec{W}_1. \quad (21)$$

Writing (21) in the form:

$$\vec{W}_2 = G\vec{W}_1; G = E + Bh; \tag{22}$$

we obtain the boundary conditions describing the effect of a thin layer with piezoelectric properties.

The condition λ >>h makes it possible to exclude the construction of a wave field inside a thin layer, in view of the quasi-static nature of the loaded state. Boundary conditions (22) are the desired conditions for non-rigid contact. For h →, from (22) the hard contact condition (1) follows. The influence of the physical and mechanical properties of a thin layer is taken into account by the elements of matrix *B*. Similarly to condition (8), the influences of other thin layers can be taken into account. As follows from (21), for this it is necessary and sufficient to know the matrix *B* (19) or the system of equations (18).

If in the domain $z \in [0, H]$ the left boundary of the thin layer is at $z = z_1$, then the layer matrix has the form:

$$T(0,H) = T_2 G T_1; \tag{23}$$

T_1 – layer matrix $[0; z_1]$, T_2 – layer matrix $[z_1, H]$.

If there are N thin layers inside some layer, the matrix of the total layer is written as:

$$T = T_{N+1} G_N T_N G_{N-1} ... T_2 G_1 T_1. \tag{24}$$

At present, the matricant method has been developed and equations of the type (18) have been obtained for piezoelectric and piezomagnetic media, considering the magnetoelectric effect, elastic, thermoelastic, liquid crystal media. Based on the boundary conditions in the system (18) for these media, it is possible to design various artificial heterostructures by introducing thin layers with different physical and mechanical properties.

An important aspect of the application of boundary conditions of the type (22) is considering the influence of deformation and distortion of crystalline media in contact with different lattice periods, as well as the study of contact distortions and their influence on physical and mechanical parameters.

One of the design features of the matricant method is the possibility, within the framework of this method, to investigate the δ - shaped properties of the medium. These properties simulate the case of a significant difference between the properties of a thin layer and the properties of the environment. Mathematically, this is written as follows:

$$\lim_{h \to 0} Bh = G; \lim_{h \to 0} b_{ij} h = g_{ij}$$

Conclusions

In this article, based on the analytical method of the matricant, the regularities of the propagation of electro-elastic waves in piezo crystals of the rhombic syngony of class 222 are studied. The complete system of Maxwell's equations and equations of motion are obtained and solved. A generalization of non-classical contact conditions for studying the effect of thin interlayers with different physical and mechanical properties on wave processes is given. The derivation of these conditions for a thin layer with piezoelectric properties is given. The possibility of studying layers with δ - shaped properties (- Dirac function) is proved.

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Н.А. Испулов, Абдул Кадыр, А.Ж. Жумабеков, А.А. Курманов, Қ.Р. Досумбеков

Анизотропты ортада толқындардың таралуы кезінде физика-механикалық қасиеттері әртүрлі жұқа қабаттардың классикалық емес шекаралық байланыс шарттары туралы

Толқындық процестер физиканың әртүрлі салаларында қарқынды зерттеледі: электродинамика, плазма физикасы, радиофизика, акустика, гидродинамика және т.б. Электромагниттік және серпімді толқындық процестерді зерттеумен қатар, өзара трансформация болған кезде әртүрлі физикалық сипаттағы толқындардың таралу заңдылықтарын зерттеу ерекше өзекті болып табылады. Байланысқан өрістердегі толқындық процестер серпімді, электромагниттік және жылу өрістерінің өзара әсерін көрсетеді. Электромагниттік өрістердің деформация өрісімен байланысы пьезоэлектрлік, пьезомагниттік және магнитострикциялық қасиеттері бар ортада жүреді. Мақалада матрицалық әдіс негізінде физика-механикалық қасиеттері әртүрлі орталарда байланысқан серпімді және электромагниттік толқындардың таралуы зерттелген. Авторлар толқын процестеріне физика-механикалық қасиеттері әртүрлі жұқа қабаттардың әсерін зерттеу үшін классикалық емес байланыс жағдайларын жалпылауды ұсынған. 222 класындағы ромбтық жүйенің анизотропты орталарында электрлік серпімді толқындардың таралуын сипаттайтын айнымалы коэффициенттері бар 1-ші ретті дифференциалдық теңдеулер жүйесі құрылды. Пьезоэлектрлік қасиеттері бар жұқа қабат үшін қатты емес байланыс жағдайларының нәтижесі келтірілген. δ — тәрізді қасиеттері бар қабаттарды зерттеу мүмкіндігі дәлелденді (δ — Дирак функциясы).

Кілт сөздер: Максвелл теңдеулері, анизотропты орта, толқындар, қатты емес байланыс, матрицант әдісі.

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О неклассических граничных условиях контакта тонких прослоек с различными физико-механическими свойствами при распространении волн в анизотропных средах

Волновые процессы интенсивно изучены в различных областях физики: электродинамике, физике плазмы, радиофизике, акустике, гидродинамике и т.д. Наряду с изучением электромагнитных и упругих волновых процессов особую актуальность приобретают исследования закономерностей распространения волн различной физической природы при наличии взаимной трансформации. Волновые процессы в связанных полях отражают взаимовлияние упругих, электромагнитных и тепловых полей. Связанность электромагнитных полей с полем деформаций имеет место в среде с пьезоэлектрическими, пьезомагнитными и магнитострикционными свойствами. В статье, на основе матричного метода, исследовано распространение связанных упругих и электромагнитных волн в средах с различными физико-механическими свойствами. Авторами предложено обобщение неклассических условий контакта для исследования влияния тонких прослоек с различными физико-механическими свойствами на волновые процессы. Построена система дифференциальных уравнений 1-го порядка с переменными коэффициентами, описывающими распространение электроупругих волн в анизотропных средах ромбической системы класса 222. Приведен вывод условий нежесткого контакта для тонкого слоя с пьезоэлектрическими свойствами. Доказана возможность исследования слоев с -образными свойствами (δ-функция Дирака).

Ключевые слова: уравнения Максвелла, анизотропная среда, волны, нежесткий контакт, метод матрицанта.

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