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## МАЗМУНЫ

### МАТЕМАТИКА

<i>Әбиев Н.А., Шонтаева Ж.С.</i> Сингулярлы ауытқыған сызықсыз интегро-дифференциалдық теңдеулер жүйесінің периодты шешімінің бар болуы және жалғыздығы .....	4
<i>Әбиев Н.А., Шонтаева Ж.С.</i> Сингулярлы ауытқыған сызықсыз интегро-дифференциалдық теңдеулер жүйесінің периодты шешімінің асимптотикалық жіктелінуі .....	9
<i>Әлібиев Д.Б., Жұмаиева А.Т., Қажыкенова А.Ш., Сейітімбетова Ә.Б.</i> Бұлтты технологияларды білім жүйесінде пайдалану .....	13
<i>Әлібиев Д.Б., Қажыкенова А.Ш., Сексембаева М.Ә.</i> Windows 7 үшін есептеу жүйелерін тиімдеуді зерттеу .....	19
<i>Әлібиев Д.Б., Сұлтанова Г.А.</i> Кластар диаграммасын жобалауда қолдану .....	24
<i>Әміров Ә.Ж., Баймолдин М.Қ., Шакирова Ю.К., Әбілдаева Г.Б., Савченко Н.К.</i> Enterprise Resource Planning — кәсіпорынның ресурстарын жобалау жүйелері .....	30
<i>Антипов Ю.Н., Омаров Г.Т., Шаяхметова Б.Қ.</i> Құрылымдық бағдарламалауды оқыту әдістемесі .....	36
<i>Аринов Е., Испулов Н.А.</i> Терең орналасқан сфера түріндегі тау-кен тұтқыр маңайындағы кернеулердің шоғырлануы .....	41
<i>Допира Р.И.</i> Web-қосымшаның стилін құру ....	45
<i>Ешкеев А.Р.</i> Йонсондық жиындар және олардың әр түрлі теориялық-модельдік қасиеттері ..	53
<i>Ешкеев А.Р., Жолмағамбетова Б.Р.</i> Позитивті йонсон теориясы экзистенциалды-тұйық моделі класындағы алгебралық шығынқы позитивті қарапайым моделі .....	63
<i>Ешкеев А.Р., Қағазбаев Ж.А.</i> Математика мамандығының магистранттары мен студенттердің деңгейін көтеруде көптілділіктің рөлі .....	70
<i>Қажыкенова С.Ш., Оспанова Б.Р.</i> Қазақ тіліндегі әр стильді мәтіннің ақпараттық талдауы ..	74
<i>Қажыкенова А.Ш., Тұрдыбекова К.М.</i> Цезийдің тұтқырлығын кристалды жылжымалы бөлшектердің кластерлік ассоциациясын ескеріп температуралық тәуелділігін талдау .....	80
<i>Мақажанова Т.Х.</i> Ақырлы өлшемді кеңістіктердің ішкі кеңістіктерінің дөңестігі мен жүлдыздылығы туралы .....	86

## СОДЕРЖАНИЕ

### МАТЕМАТИКА

<i>Абиев Н.А., Шонтаева Ж.С.</i> Существование и единственность периодического решения системы нелинейных сингулярно-возмущенных интегро-дифференциальных уравнений .....	4
<i>Абиев Н.А., Шонтаева Ж.С.</i> Асимптотическое разложение периодического решения системы сингулярно-возмущенных нелинейных интегро-дифференциальных уравнений .....	9
<i>Алибиев Д.Б., Жумаиева А.Т., Кажикенова А.Ш., Сейтімбетова А.Б.</i> Использование облачной технологии в образовании .....	13
<i>Алибиев Д.Б., Кажикенова А.Ш., Сексембаева М.А.</i> Исследование оптимизации вычислительных систем для Windows 7 .....	19
<i>Алибиев Д.Б., Сұлтанова Г.А.</i> Использование диаграмм классов в планировании .....	24
<i>Амиров А.Ж., Баймұльдин М.Қ., Шакирова Ю.К., Абилдаева Г.Б., Савченко Н.К.</i> Enterprise Resource Planning — системы планирования ресурсов предприятия .....	30
<i>Антипов Ю.Н., Омаров Г.Т., Шаяхметова Б.К.</i> Методика преподавания структурного программирования .....	36
<i>Arinov E., Ispulov N.A.</i> The voltage concentration in the vicinity of spherical mine workings of deep foundations .....	41
<i>Допира Р.И.</i> Разработка стиля Web-приложения .....	45
<i>Ешкеев А.Р.</i> Йонсоновские множества и их некоторые теоретико-модельные свойства .....	53
<i>Ешкеев А.Р., Жолмағамбетова Б.Р.</i> Позитивно алгебраически простые модели в классе экзистенциально-замкнутых моделей выпуклых позитивных йонсоновских теорий .....	63
<i>Ешкеев А.Р., Қағазбаев Ж.А.</i> Роль полиязычия в повышении уровня подготовки магистрантов и студентов математических специальностей .....	70
<i>Қажикенова С.Ш., Оспанова Б.Р.</i> Информационный анализ текстов различных стилей на казахском языке .....	74
<i>Қажикенова А.Ш., Тұрдыбекова К.М.</i> Анализ вязкости цезия в зависимости от температуры с учетом ассоциации кластеров из кристаллоподвижных частиц .....	80
<i>Мақажанова Т.Х.</i> О выпуклости и звездности подмножеств в конечномерных пространствах ...	86

<i>Нұрғабұлылов Е.Д., Нұрғабұлы Д.Н.</i> Ерекше ауытқыған шекаралық есептің шекаралық секіріс құбылысы туралы .....	91	<i>Нургабылов Е.Д., Нургабыл Д.Н.</i> Об одном явлении граничного скачка сингулярно возмущенной краевой задачи.....	91
<i>Попова Н.В., Базикова К.М., Есендаулетова Ж.Т.</i> «Операцияларды зерттеу» пәніне арналған электрондық оқулықты әзірлеу және қолдану .....	95	<i>Попова Н.В., Базикова К.М., Есендаулетова Ж.Т.</i> Разработка и применение электронного учебника по дисциплине «Исследование операций».....	95
<i>Рамазанов М.Ы., Жанболова А.К.</i> Аралас типті жүктеулі теңдеу үшін берілген бір шеттік есеп туралы .....	100	<i>Ramazanov M.I., Zhanbolova A.K.</i> About a boundary value problem for the loaded equation of mixed type.....	100
<i>Серік М., Бақиев М.Н., Нұрбекова Г.Ф.</i> Сызық бойынша MINDSTORMS NXT роботының қозғалу программасын жазуға әдістемелік нұсқау .....	107	<i>Серик М., Бакиев М.Н., Нурбекова Г.Ф.</i> Методические указания по разработке программы движения робота MINDSTORMS NXT вдоль линии .....	107
<i>Серік М., Байғараева А.Е.</i> Параллель есептеулер кластерін баптау.....	112	<i>Серик М., Байгараева А.Е.</i> Настройка кластера параллельных вычислений .....	112
<i>Сұлтанов М.А.</i> Қисынсыз Коши есебі үшін қосқабатты айырымдық схемалардың орнықтылығы.....	118	<i>Султанов М.А.</i> Об устойчивости двухслойных разностных схем для некорректной задачи Коши.....	118
<i>Сұлтанов М.А., Сағынбекова Э.С., Марасулов А.М.</i> Алгоритмдеу процесін оқытуды қолдаушы электрондық орталарды құру технологиялары .....	123	<i>Султанов М.А., Сагинбекова Э.С., Марасулов А.М.</i> Технологии разработки электронных сред поддержки обучения процесса алгоритмизации .....	123
<i>Тілеуқенов С.Қ., Аринов Е., Испұлов Н.А., Сейітханова А.Қ.</i> 222 және mm2 класты ромбылық сингониялы анизотропты орталарда жылулық және серпімді толқындардың өшу және жылдамдық коэффициенттері.....	129	<i>Tleukenov S.K., Arinov E., Ispulov N.A., Seitkhanova A.K.</i> The attenuation coefficient and the velocity of thermal and elastic waves in orthorhombic syngony anisotropic media classes 222 and mm2 .....	129
<i>Төреходжаев Ә.Н., Маматова Г.Ө., Рыстыгулова В.Б.</i> Серпімді біртекті емес пластинаның әрқелкі температуралық өрістегі күрделі иілуі .	135	<i>Туреходжаев А.Н., Маматова Г.У., Рыстыгулова В.Б.</i> Сложный изгиб упругой неоднородной пластины в неравномерном температурном поле .....	135
<i>Омарова А.Т., Грело М.Ф.</i> Экономиканың индустриалды-инновациялық бағытын талдау Қазақстан Республикасының дамуының жаңа векторы ретінде .....	141	<i>Омарова А.Т., Грело М.Ф.</i> Анализ индустриально-инновационной ориентации экономики как нового вектора развития Республики Казахстан .....	141
<i>Орумбаева Н.Т.</i> Гиперболалық теңдеулер жүйесінің периодты шешімдері .....	150	<i>Орумбаева Н.Т.</i> Периодические решения систем гиперболических уравнений.....	150
АВТОРЛАР ТУРАЛЫ МӘЛІМЕТТЕР.....	155	СВЕДЕНИЯ ОБ АВТОРАХ.....	155

S.Tleukenov<sup>1</sup>, E.Arinov<sup>2</sup>, N.A.Ispulov<sup>3</sup>, A.K.Seytkhanova<sup>3</sup>

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## **The attenuation coefficient and the velocity of thermal and elastic waves in orthorhombic syngony anisotropic media classes 222 and mm2**

The relevance of research of wave propagation patterns in elastic medium with thermomechanical effect is related to the need to solve theoretical and applied problems of geophysics, seismology, mechanics of composite materials, etc. Bound equations of motion and heat conduction equation differ by complexity and an abundance of physical and mechanical parameters. In connection with this there is a rapidly developing branch of mechanics of deformable solids - thermoelasticity. Within this framework, based on the use of certain physical and mechanical properties of anisotropic medium, we study bound thermal and mechanical fields. In this paper, based on the method of matriciant, we identify the types of dependencies of velocities and attenuation coefficients of bound thermoelastic waves of frequency; high-quality graphics of velocities and damping coefficients of frequency are constructed under changing the parameters of the medium (thermo-mechanical parameters, temperature and thermal conductivity).

*Key words:* Anisotropic medium, thermoelasticity, Fourier heat equation, harmonic waves, dispersion, periodic structure, matriciant.

### *Introduction*

The dynamical theory of thermoelasticity is the study of dynamical interaction between thermal and mechanical fields in solid bodies and is of much importance in various engineering fields such as earthquake engineering, soil dynamics, aeronautics, nuclear reactors, etc. It is well known that the classical theory of thermoelasticity [1, 2] rests upon the hypothesis of the Fourier law of heat conduction, in which the temperature distribution is governed by a parabolic-type partial differential equation. The theory predicts that a thermal signal is felt instantaneously everywhere in a body. This is unrealistic from the physical point of view, especially for short-time responses. To account for the effect of thermal relaxation, generalized thermoelasticity has been formulated on the basis of a modified Fourier law such that the temperature distribution is governed by a hyperbolic-type equation. Accordingly, heat transport in solids is regarded as a wave phenomenon rather than a diffusion phenomenon.

The wave propagation in anisotropic inhomogeneous medium is considered. A new method of matriciant has been developed. The method of matriciant allows to investigate wave processing in anisotropic medium with various physical and mechanical properties [3–5].

The structure of matriciant for the equation motion elastic media equations, equations of thermo-mechanical medium has been established. Wave propagation in infinite and finite periodical inhomogeneous media are studied.

The application of matriciants method for non-destructive testing and wave propagation in thermo elastic media is considered [6].

In the paper [7], waves propagating along an arbitrary direction in a heat conducting orthotropic thermoelastic plate are presented by utilizing the normal mode expansion method in generalized theory of thermoelasticity with one thermal relaxation time. In the paper [8], authors studied the interaction of free harmonic waves with multilayered media in generalized thermoelasticity by utilizing the combination of the linear transformation formation and transfer matrix method approach. Solutions obtained are general and pertain to several special cases. Of these mention: (a) dispersion characteristics for a multilayered.

### *A Matriciant Method*

At the present days solving wide range theoretical and applied problems of continuum dynamics requires more thorough consideration of anisotropy and physical and mechanical properties. The main peculiarity of analyzing wave processes in anisotropic medium is inapplicability of physical interpretations and mathematical methods developed for isotropic medium. It is related to the fact that it is impossible to sepa-

rate wave field to forward and back waves. The other essential difficulty is an existence of a lot of physical parameters.

The method of study is an analytical method based on developing matrix techniques to study dynamics of the elastic layered medium.

The main idea is to deduce initial equations of the continuous medium and equations describing wave propagation in medium, based on the method of separation of variables, (solutions are represented as plane waves) to the equivalent set of ordinary differential equations with variable coefficients and then build the structure of a Matriciant (normalized matrix of fundamental solutions).

The problems of wave propagation in anisotropic medium, propagation of electromagnetic, electro-elastic, piezoelectric waves in anisotropic dielectrics, propagation of waves in anisotropic elastic and thermo-elastic medium, propagation of waves in anisotropic dielectric medium with magneto-electric effects, and orthotropic planes are analyzed by using a matriciant method.

The main advantage of a Matriciant method is equality of describing wave processes under the presence of one or several physical effects: elastic, thermo-elastic, magneto-elastic, piezo-elastic and magneto-electric, piezo-magnetic and magneto-electric effects.

In S.K. Tleukenov's international publications, the structure of the equations of motion matriciants in inhomogeneous medium were defined [6–8]. These publications were the beginning of a completely new level of studying the dynamics of inhomogeneous medium with application of that method and corporate studying of waves different by nature in inhomogeneous and periodically inhomogeneous anisotropic medium.

Consequently, development of the studying techniques and constituting interpretations about wave behavior in anisotropic medium should be considered as one of the high priority problem in theoretical physics and mechanics of deformable solids.

*The matrix formulation of the propagation of thermoelastic waves.*

Propagation of thermoelastic waves in anisotropic media described by the equations of motion to be solved together with the Fourier heat equation and the equation of heat flow, which have the form:

$$\sigma_{ij,j} = \rho \ddot{U}_i, \quad (1)$$

$$\lambda_{ij} \frac{\partial \theta}{\partial x_j} = -q_i, \quad (2)$$

$$\frac{\partial q_i}{\partial x_i} = -i\omega \beta_{ij} \varepsilon_{ij} - i\omega \frac{c_\varepsilon}{T_0} \theta, \quad (3)$$

where  $\sigma_{ij}$  — stress tensor,  $\rho$  — density of the medium;  $\lambda_{ij}$  — thermal conductivity tensor;  $q_i$  — the vector of heat;  $\omega$  — the angular frequency;  $\beta_{ij}$  — thermomechanical constants,  $\beta_{ij} = \beta_{ji}$ ;  $\varepsilon_{ij}$  — the strain tensor,  $c_\varepsilon$  — specific heat at constant strain;  $\theta = T - T_0$  — temperature increase compared with the temperature of the natural state  $T_0$ ,  $\left| \frac{\theta}{T_0} \right| \ll 1$  for small deformations.

Physical and mechanical quantities are related by relation of Duhamel-Neumann:

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - \beta_{ij} \theta. \quad (4)$$

Here  $c_{ij}$  — the elastic parameters;  $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$ ;  $\varepsilon_{kl}$  — the tensor Cauchy for small deformations.

Equations (1)–(4) determine the relationship of mechanical stress and temperature as a function of the independent variables — the thermal field and deformation.

Thus, the relation (1)–(4) constitute a closed system of thermoelasticity equations, which describes the propagation of thermoelastic waves.

Based on the method of separation of variables in the case of a harmonic function of time:

$$\left[ U_i(x, y, z, t); \sigma_{ij}(x, y, z, t); \theta; q_z \right] = \left[ U_i(z), \sigma_{ij}(z), \theta; q_z \right] e^{i(\omega t - mx - ny)}. \quad (5)$$

The system of equations (1)–(4) reduces to a system of differential equations of first order with variable coefficients which describes the propagation of harmonic waves:

$$\frac{d\vec{W}}{dz} = B\vec{W}, \quad (6)$$

here  $B = B[c_{ijkl}(z), \beta_{ij}(z), \omega, m, n]$  — coefficient matrix whose elements contain the parameters of the medium in which waves propagate thermo elastic;  $m, n$ -components of the wave vector  $\tilde{k}$ .

The vector  $\vec{W}$  has the form:

$$\vec{W}(x, y, z, t) = [u_z(z), \sigma_{zz}, u_x(z), \sigma_{xz}, u_y(z), \sigma_{yz}, \theta, q_z]^t \exp(i\omega t - imx - iny). \quad (7)$$

The symbol  $^t$  indicates the transpose of the vector — a vector of strings — Column.

The heterogeneity of the medium is assumed along  $Z$ . In constructing the coefficient matrix  $B$  is used as a representation of the solution (5), the system of equations (1)–(4) are in the derivatives along the coordinate  $Z$  and the excluded components of the stress tensor is not included in the boundary conditions. The multiplier  $\exp(i\omega t - imx - iny)$  is omitted throughout.

### Solution of the problem

In the case of one dimensional thermoelastic wave propagation in orthorhombic syngony anisotropic medium coefficients matrix  $B$  (if medium parameters are constant) has the following form:

$$B_0 = \begin{pmatrix} 0 & b_{12} & b_{17} & 0 \\ b_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{78} \\ 0 & -i\omega b_{17} & b_{87} & 0 \end{pmatrix}, \quad (8)$$

here, coefficients  $b_{ij}$  are given by:

$$b_{12} = \frac{1}{c_{33}}; \quad b_{17} = \frac{\beta_{33}}{c_{33}}; \quad b_{21} = -\omega^2 \rho; \quad b_{87} = -i\omega \left( \frac{\beta_{33}^2}{c_{33}} + c_\epsilon \right); \quad b_{78} = -\frac{1}{\lambda_{33}}.$$

Considering condition [5]:

$$\det|B - \lambda E| = 0, \quad (9)$$

for this problem we obtain characteristic equation of the following form:

$$\lambda^4 - B\lambda^2 + C = 0, \quad (10)$$

where  $B = b_{12}b_{21} + b_{78}b_{87}$ ,  $C = b_{21}b_{78}(i\omega b_{17}^2 + b_{12}b_{87})$

from (10) we obtain:

$$\lambda_{1,2}^2 = \frac{1}{2}(b_{12}b_{21} + b_{78}b_{87}) \pm \frac{1}{2}\sqrt{(b_{12}b_{21} - b_{78}b_{87})^2 - 4i\omega b_{17}^2 b_{21}b_{78}}. \quad (11)$$

If we concede that longitudinal elastic and heat waves propagate unbound that is thermomechanical parameters  $\beta_{ij} = 0$ , then roots of characteristic equation (3) will be equal to:

$$\lambda_{1,2} = \pm i\omega \sqrt{\frac{\rho}{c_{33}}}; \quad \lambda_{3,4} = \pm \sqrt{\frac{i\omega c_\epsilon}{\lambda_{33}}}. \quad (12)$$

The first root of the relation (12) gives velocity of longitudinal wave that propagates with attenuation; second relation determines heat wave.

From the relation (11) we get four roots of characteristic equation (10) having following form:

$$k_{1,2} = \pm \sqrt{\frac{a}{2} \left( 1 + \frac{2b-c}{\sqrt{2}\sqrt{D-x}} \right) + \frac{1}{2}i \left( b - \frac{1}{\sqrt{2}}\sqrt{D-x} \right)}; \quad k_{3,4} = \pm \sqrt{\frac{a}{2} \left( 1 - \frac{2b-c}{\sqrt{2}\sqrt{D-x}} \right) + \frac{1}{2}i \left( b + \frac{1}{\sqrt{2}}\sqrt{D-x} \right)}. \quad (13)$$

where  $a = b_{12}b_{21}$ ;  $b = b_{78}b_{87}$ ;  $c = 4i\omega b_{17}^2 b_{21}b_{78}$ ;  $D = \sqrt{(a^2 + b^2)^2 + (2ab - c)^2}$ .

These roots have already taken into account an effect that elastic and heat waves are bound that is  $\beta_{ij} \neq 0$ .

Let's rewrite  $k_{1,2}$  in (13) in the following form:

$$k_{1,2} = \pm \sqrt{x_1 + iy_1} = \sqrt[4]{x_1^2 + y_1^2} (Cos\psi + iSin\psi); \quad (14)$$

$$k_{1,2} = \pm\sqrt{x_1 + iy_1} = \pm\frac{1}{\sqrt{2}}\left(\frac{y_1}{\sqrt{D_1 + x_1}} + i\sqrt{D_1 + x_1}\right), \quad (15)$$

$$\text{where } D_1 = \sqrt{x_1^2 + y_1^2}, \quad x_1 = \frac{a}{2}\left(1 + \frac{2b - \frac{c}{a}}{\sqrt{2}\sqrt{D - x}}\right); \quad y_1 = \frac{1}{2}\left(b - \frac{1}{\sqrt{2}}\sqrt{D - x}\right).$$

Roots  $k_{3,4}$  in (13) are equal:

$$k_{3,4} = \pm\sqrt{x_2 + iy_2} = -\frac{1}{\sqrt{2}}\left(\frac{y_2}{\sqrt{D_2 + x_2}} \pm i\sqrt{D_2 + x_2}\right), \quad (16)$$

$$\text{where } x_2 = \frac{a}{2}\left(1 - \frac{2b - \frac{c}{a}}{\sqrt{2}\sqrt{D - x}}\right); \quad y_2 = \frac{1}{2}\left(b + \frac{1}{\sqrt{2}}\sqrt{D - x}\right).$$

In an explicit form roots (15) and (16) have following form:

$$k_{1,2} = \pm\sqrt{\frac{c_e \omega}{2\lambda_{33}}}(1+i)\left[1 + \frac{\lambda_{33}}{2}\left(\frac{i\omega c_e \rho \lambda_{33} T_0^2 + c_{33}^3}{\rho^2 \omega^3 \lambda_{33}^2 T_0^2 + \omega c_e^2 c_{33}^2}\right)\beta_{33}^2\right]; \quad (17)$$

$$k_{3,4} = \pm i\sqrt{\frac{\rho \omega^2}{c_{33}}}\left(1 - \frac{i\omega}{2c_{33}\lambda_{33}}\left(\frac{\rho \omega c_{33}\lambda_{33}^2 T_0^2 - i c_e c_{33}^2 \lambda_{33} T_0}{\rho^2 \omega^3 \lambda_{33}^2 T_0^2 + \omega c_e^2 c_{33}^2}\right)\beta_{33}^2\right). \quad (18)$$

Let's consider root  $k_1$  in relations (17).

Real and imaginary parts of this root are equal:

$$\text{Re } k_1 = \sqrt{\frac{c_e \omega}{2\lambda_{33}}}\left[1 + \frac{\lambda_{33}}{2}\left(\frac{c_{33}^3 - \omega c_e \rho \lambda_{33} T_0^2}{\rho^2 \omega^3 \lambda_{33}^2 T_0^2 + \omega c_e^2 c_{33}^2}\right)\beta_{33}^2\right]; \quad (19)$$

$$\text{Im } k_1 = i\sqrt{\frac{c_e \omega}{2\lambda_{33}}}\left[1 + \frac{\lambda_{33}}{2}\left(\frac{\omega c_e \rho \lambda_{33} T_0^2 + c_{33}^3}{\rho^2 \omega^3 \lambda_{33}^2 T_0^2 + \omega c_e^2 c_{33}^2}\right)\beta_{33}^2\right]. \quad (20)$$

From the imaginary part of the root  $k_1$  we obtain formula for velocity of heat wave:

$$c = \frac{k_1}{\omega} = \sqrt{\frac{2\lambda_{33} T \omega}{c_e}}\left[1 - \frac{\lambda_{33}}{2}\left(\frac{\omega c_e \rho \lambda_{33} T_0^2 + c_{33}^3}{\rho^2 \omega^2 \lambda_{33}^2 T_0^2 + c_e^2 c_{33}^2}\right)\beta_{33}^2\right]. \quad (21)$$

Real part of this root allows to get attenuation coefficient of heat wave:

$$k_{\text{sam}} = \sqrt{\frac{c_e \omega}{2\lambda_{33}}}\left[1 + \frac{\lambda_{33}}{2}\left(\frac{c_{33}^3 - \omega c_e \rho \lambda_{33} T_0^2}{\rho^2 \omega^2 \lambda_{33}^2 T_0^2 + c_e^2 c_{33}^2}\right)\beta_{33}^2\right]. \quad (22)$$

Now, let's consider positive root  $k_3$  in relation (18).

Real and imaginary parts of this root allows to get attenuation coefficient and velocity of elastic wave:

$$k_{\text{sam}} = \frac{1}{2}\sqrt{\frac{\rho}{c_{33}}}\left(\frac{\rho \omega^3 c_{33} \lambda_{33} T_0^2}{\rho^2 \omega^2 \lambda_{33}^2 T_0^2 + c_e^2 c_{33}^2}\right)\beta_{33}^2; \quad (23)$$

$$c = \sqrt{\frac{c_{33}}{\rho}}\left(1 - \frac{1}{2}\left(\frac{c_e c_{33} T_0}{\rho^2 \omega^2 \lambda_{33}^2 T_0^2 + c_e^2 c_{33}^2}\right)\beta_{33}^2\right). \quad (24)$$

As a result of the roots (17), (18) high quality graphics, presented below, of dependencies of velocity and attenuation coefficients of elastic and heat wave from frequency are constructed under, changing the parameters of the medium (thermo-mechanical parameters, temperature).

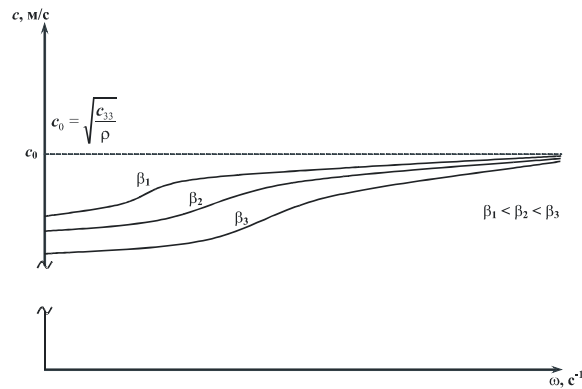


Figure 1. Diagram of velocity  $c$  of elastic longitudinal wave and frequency under different thermomechanical parameters  $\beta_{ij}$

From the given diagram it can be seen that under increase of thermomechanical parameter velocity of longitudinal elastic wave decreases.

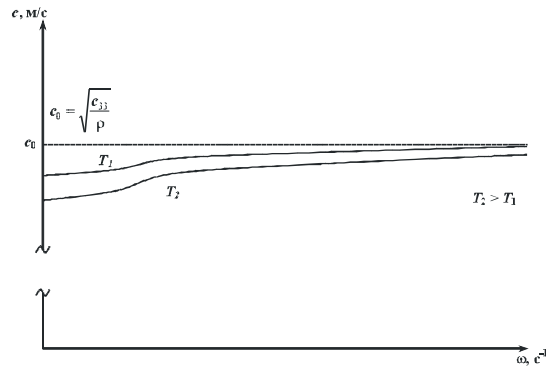


Figure 2. Elastic longitudinal wave velocity  $c$  and frequency diagram under different temperatures

This diagram indicates that an increase of thermodynamic temperature causes a decrease of velocity of elastic longitudinal wave. It's related to lattice site oscillation that affects wave velocity.

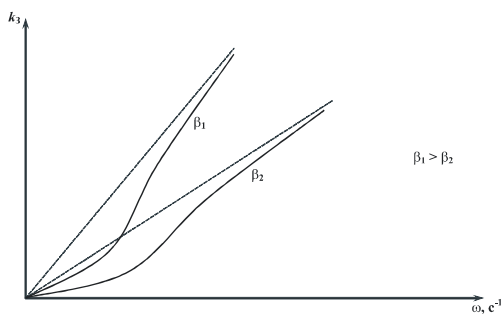
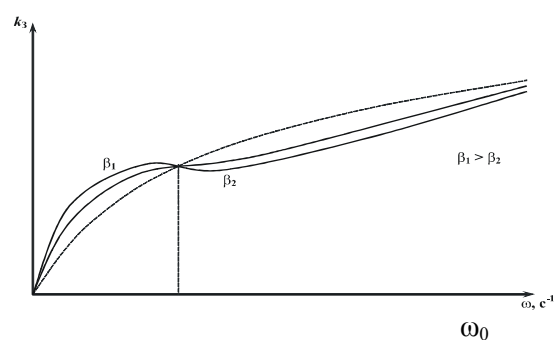


Figure 3. diagram of attenuation coefficient  $k_3$  of elastic longitudinal wave and frequency under different thermomechanical parameters  $\beta_{ij}$



Picture 4. diagram of attenuation coefficient  $k_3$  of heat wave and frequency under different thermomechanical parameters  $\beta_{ij}$

It follows from the last diagram that an increase of thermomechanical parameter causes attenuation of heat wave in anisotropic medium. Under explicit magnitude of frequency  $\omega_0$  which can be derived from equation (22) there is no interaction of heat and elastic waves that is these waves propagate without thermoelastic effect and this frequency will be valid under any thermomechanical parameter  $\beta_{ij}$ .



### Conclusion

In this paper, based on the method of matrixant, we identify the types of dependencies of velocities and attenuation coefficients of bound thermoelastic waves of frequency; high-quality graphics of velocities and damping coefficients of frequency are constructed under changing the parameters of the medium (thermo-mechanical parameters, temperature and thermal conductivity).

### References

- 1 Новацкий В. Динамические проблемы термоупругости. — Нидерланды, 1975. — 550 с.
- 2 Новацкий В. Теория упругости. — М.: Мир, 1986. — 556 с.
- 3 Tleukenov S. Investigation of the thin layer influence of the boundary conditions. Abstracts «Seminar on earthquake processes and their consequences». — Kurukshetra: India, 1989. — P. 4.
- 4 Tleukenov S. The structure of propagator matrix and its application in the case of the periodical inhomogeneous media. Abstr. Semin. on Earthquake processes and their consequences Seismological investigations. — 1989. — Kurukshetra, India. — P. 2–4.
- 5 Тлеуєнов С.К. Метод матрицанта. — Павлодар: НИЦ ПГУ им. С.Торайгырова, 2004. — 148 с.
- 6 Неразрушающий контроль: Справочник: В 7 т. / Под общ. ред. В.В. Клюева. — Т. 4: В 3 кн. Кн. 1: Акустическая тензометрия / В.А. Анисимов, Б.И. Каторгин, А.Н. Куценко и др. — М.: Машиностроение, 2004. — 736 с.: ил.
- 7 Verma K.L. Thermoelastic Waves in Anisotropic Plates using Normal Mode Expansion Method with Thermal Relaxation Time // *International Journal of Aerospace and Mechanical Engineering* 2:2. — P. 86–93, 2008.
- 8 Verma K.L. The general problem of thermoelastic wave propagation in multilayered anisotropic media with application to periodic media // *International Journal of Applied Engineering Research*, Dindigul Volume 1. — № 4. — P. 908–922, 2011.
- 9 Гантмахер Ф.П. Теория матриц. — М.: Наука, 1988. — 552 с.
- 10 Marshall Carleton. Pease Methods of Matrix Algebra. М.: Academic Press. — 1965.
- 11 Ашкфорт Н., Мермин Н. Физика твердого тела. — М.: Мир, 1979.

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## 222 және $mm2$ класты ромбылық сингониялы анизотропты орталарда жылулық және серпімді толқындардың өшу және жылдамдық коэффициенттері

Термомеханикалық эффектпен болатын серпімді орталарда толқындық процестердің заңдылықтарды зерттеу өзектілігі, геофизика, сейсмология, композиттік материалдардың механикасының теориялық және қолданбалы есептерді шешуінде қажеттілігімен байланысты. Байланысқан қозғалыс теңдеулері мен жылуөткізгіштік теңдеулері физика-механикалық параметрлердің күрделілігі мен көп болуымен ерекшеленеді. Осыған байланысты деформацияланатын қатты дене механикасының «Термосерпімділік» деген тарауы қарқынды дамып келеді. Осы бағыттың аясында анизотропты орталардың кейбір физика-механикалық қасиеттерін қолдана отырып, байланысқан жылулық және механикалық өрістер зерттелді. Мақалада, матрицант әдісінің негізінде, жиілікке тәуелді байланысқан термосерпімді толқындардың жылдамдықтары мен өшу коэффициенттерінің тәуелділіктердің түрлері анықталды; серпімді және жылу толқындардың (термомеханикалық параметрлердің аздығы кезіндегі) жылдамдықтардың және өшу коэффициенттерінің температураның, жылуөткізгіштік коэффициентінің және жиіліктің өзгерісіне тәуелділігінің сапалы графиктері сызылды.

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## Коэффициенты затухания и скорости тепловых и упругих волн в анизотропной среде ромбической сингонии классов 222 и $mm2$

Актуальность исследования закономерностей волновых процессов в упругих средах с термомеханическим эффектом связана с необходимостью решения теоретических и прикладных задач геофизики, сейсмологии, механики композитных материалов и т.д. Связанные уравнения движения и уравнения теплопроводности отличаются сложностью и обилием физико-механических параметров. В связи с этим интенсивно развивается раздел механики деформируемого твердого тела «Термоупругость». В рамках этого направления, опираясь на использование определенных физико-механических свойств в анизотропных средах, изучаются связанные тепловые и механические поля. В статье, на основе метода матрицанта, определены виды зависимостей скоростей и коэффициентов затухания связанных термоупругих волн от частоты; построены качественные графические зависимости скоростей и коэффициентов затухания упругих и тепловых волн от частоты при изменении параметров среды (термомеханического параметра, температуры и коэффициента теплопроводности).

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