Action of Moving Load on a Two-Layer Shell in Elastic Medium

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Abstract. Based on the stationary solution of the problem of the action of a uniformly moving load on a two-layer cylindrical shell in an unbounded elastic medium (mass), the influence of its outer layer on the values of the critical load velocities and the reaction of an elastic medium rigidly coupled to it is studied. The speed of the load movement is assumed to be subsonic (less than the speed of propagation of shear waves in the elastic medium and the outer layer of the shell). The need for research is due to the fact that the design scheme widely used in the dynamic calculation of a deep tunnel - a homogeneous (single-layer) shell in an elastic space, in some cases is not adequate to the considered underground structure. The inner (bearing) layer of the considered shell is a thin-walled shell, rigidly coupled to its outer (enclosing) layer of arbitrary thickness. To describe the motion of the massif and the outer (enclosing) layer of the shell, the dynamic equations of the theory of elasticity in Lame potentials are used. Oscillations of the inner (bearing) layer of the shell are described by the classical equations of shell theory based on the Kirchhoff-Love hypotheses. The equations are presented in a moving cylindrical coordinate system associated with a moving load. When solving the problem, the integral Fourier transform along the axial coordinate is used, which makes it possible to consider the load distributed along the shell axis according to an arbitrary law. When carrying out numerical experiments, the load moving at a given speed was assumed to be uniformly distributed in a certain area along the lower half of the inner surface of the shell. The outer (enclosing) layer was assumed to have different rigidity in relation to the rigidity of the array. The calculation results presented in the form of tables and graphs are analyzed in detail. From the analysis of the calculation results, it follows that the enclosing layer, as well as its rigidity, to a large extent affects both the critical load speeds and the displacements and stresses in the array.

Keywords: Tunnel · Elastic space · Two-layer shell · Moving load · Critical speed · Stress-strain state

1 Introduction

Experimental studies show that when a transport load (load from a moving intra-tunnel transport) acts on tunnels, vibrations occur both in the structures themselves and in the surrounding rock mass. Exceeding the permissible levels of vibrations can lead to a loss

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of the bearing capacity of structures or their unsuitability for normal operation, and if they are shallow, to the same consequences for nearby ground structures [1].

It should be noted that experimental methods for studying vibration processes that occur in these structures due to the action of a transport load require significant material costs, and in some cases their implementation is not possible. In this regard, effective methods for their dynamic calculations are needed, based on mathematical models using modern concepts of mechanics.

As the main model problems used to study the dynamics of tunnels under the influence of transport load, problems are usually considered about the action on a circular cylindrical shell located in an elastic space or half-space, which uniformly moves along its inner surface along the generatrix. The first task simulates the dynamic behavior of a deep structure, the second – a shallow one.

Problems for an elastic half-space are more complex than for an elastic space, since it becomes necessary to take into account the waves reflected by the boundary of the half-space. Therefore, the number of publications devoted to the study of this problem is not numerous and covers mainly recent years [2–10]. In these works, when constructing a mathematical model, the tunnel lining was considered as a homogeneous elastic circular cylindrical shell.

The problems of the action of moving axisymmetric loads on a thin-walled and thick-walled circular cylindrical shell in an elastic medium were considered in the articles by V.I. Pozhuev "The action of a moving load on a cylindrical shell in an elastic medium", 1978; V.M. Lvovsky et al. "Steady-state vibrations of a cylindrical shell in an elastic medium under the action of a moving load", 1974; [11]. Similar problems under the action of non-axisymmetric moving loads on the shell were solved in [2, 3, 12, 13]. These problems are model in the study of the dynamics of deep tunnels, supported by a homogeneous cylindrical shell (lining), under the influence of a traffic load. However, the use of such a model of deep tunnels can be limited, for example, when tunneling by drilling and blasting, when the solidity of the massif is broken, technological fracturing appears and the actual shape of the tunnel deviates from the design one. To eliminate voids and close contact of the lining with the massif, the space behind the lining is cemented or packed (filled with bulk material). The layer created in this way, which separates the lining from the rock mass and has physical and mechanical characteristics that are different from it, as well as from the lining material, must be taken into account when constructing a mechanical and mathematical model of an underground structure, considering the lining and the artificially created layer as a two-layer shell. In addition, to dampen vibrations in the rock mass arising from loads moving in the tunnel, an additional layer of various rocks enclosing the lining from the rock mass can be added to the tunnel structure. In this case, the lining and the layer surrounding it can also be considered as a two-layer shell. The need to use a model in the form of a two-layer circular shell to study the dynamics of tunnels is also caused by the use of layered (for example, steel concrete) linings in the construction practice.

The action of moving periodic loads on a two-layer and multilayer circular cylindrical shell in an elastic space was studied respectively in articles by [14, 15]. In contrast to these works, in this article, the load moving along the inner surface of a two-layer shell is an aperiodic arbitrary type.

2 Materials and Methods

When solving the problem, the method of mathematical modeling was used together with the models of the theory of elasticity.

3 Results

3.1 Statement and Analytical Solution of the Problem

Let us consider a cylindrical cavity with radius R1 in an infinite linearly elastic homogeneous and isotropic medium. The cavity is supported by a two-layer shell, the inner layer of which is a thin-walled shell of thickness h0 and the radius of the middle surface R2, and a thick-walled shell is the outer layer. Due to the small thickness of the inner layer, it can be assumed that it is in contact with the outer layer along its middle surface. Load P moves forward along the inner surface of the shell in the direction of its Z axis at a constant speed c (less than the speed of propagation of shear waves in the outer layer of the shell and its environment).

To describe the motion of the inner layer of the shell, let us use the classical equations of the theory of thin shells (1), and to describe the motion of its outer layer and the environment let us use the dynamic equations of the theory of elasticity (2):

$$\frac{\partial^2 u_{0z}}{\partial z^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0z}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial z\partial\theta}+\frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial z}$$
$$=\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0z}}{\partial t^2}+\frac{1-\nu_0}{2\mu_0 h_0}(P_z-q_z),$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0z}}{\partial z\partial\theta}+\frac{(1-\nu_0)}{2}\frac{\partial^2 u_{0\theta}}{\partial z^2}+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial u_{0r}}{\partial\theta}$$
$$=\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0\theta}}{\partial t^2}+\frac{1-\nu_0}{2\mu_0 h_0}(P_\theta-q_\theta),$$
$$\frac{\nu_0}{R}\frac{\partial u_{0z}}{\partial z}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r}+\frac{u_{0r}}{R^2}==-\rho_0\frac{1-\nu_0}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial t^2}-\frac{1-\nu_0}{2\mu_0 h_0}(P_r-q_r), \tag{1}$$

where u0z, u0θ, u0r are the displacements of the points of the middle surface of the inner layer in the direction of the axes of the cylindrical coordinate system Z, θ, r; Pz, Pθ, Pr are the components of the intensity of the moving load P; at r = R2 qz = σrz2, qθ = σrθ2, qr = σrr2 are the outer layer reaction components; σrj2 are the stress tensor components in the outer layer (j = z, θ, r); ν0, μ0, ρ0 are, respectively, Poisson's ratio, shear modulus, and density of the material of the inner layer; R = R2;

$$(\lambda_k+\mu_k)grad\ div\,\mathbf{u}_k+\mu_k\nabla^2\mathbf{u}_k=\rho_k\frac{\partial^2\mathbf{u}_k}{\partial t^2},\quad k=1,\ 2. \tag{2}$$

Here and below, index 1 refers to the medium, and 2 to the outer layer of the shell; λk = 2μkνk /(1 – 2νk), μk are shear moduli, νk are Poisson's ratios, ρk are densities, uk are displacement vectors of points in space and outer layer, ∇^2 is the Laplace operator.

Since a steady process is considered, the deformation pattern is stationary with respect to the moving load. Therefore, it is convenient to pass to the moving coordinate system η = z – ct, θ, r.

Then Eqs. 1 and 2 will be rewritten in the form:

$$\left[1-\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\right]\frac{\partial^2 u_{0\eta}}{\partial\eta^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0\eta}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial\eta\partial\theta}$$
$$+\frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial\eta}=\frac{1-\nu_0}{2\mu_0 h_0}(P_\eta-q_\eta),$$
$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\eta}}{\partial\eta\partial\theta}+\frac{(1-\nu_0)}{2}\left(1-\frac{\rho_0 c^2}{\mu_0}\right)\frac{\partial^2 u_{0\theta}}{\partial\eta^2}+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}$$
$$+\frac{1}{R^2}\frac{\partial u_{0r}}{\partial\theta}=\frac{1-\nu_0}{2\mu_0 h_0}(P_\theta-q_\theta), \tag{3}$$

$$\frac{\nu_0}{R}\frac{\partial u_{0\eta}}{\partial\eta}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r}+\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial\eta^2}+\frac{u_{0r}}{R^2}=-\frac{1-\nu_0}{2\mu_0 h_0}(P_r-q_r);$$
$$\left(\frac{1}{M_{pk}^2}-\frac{1}{M_{sk}^2}\right) grad\ div\,\mathbf{u}_k+\frac{1}{M_{sk}^2}\nabla^2\mathbf{u}_k=\frac{\partial^2\mathbf{u}_k}{\partial\eta^2},\quad k=1,\ 2, \tag{4}$$

where Mpk = c/cpk, Msk = c/csk are the Mach numbers; $c_{pk}=\sqrt{(\lambda_k+2\mu_k)/\rho_k}$, $c_{sk}=\sqrt{\mu_k/\rho_k}$ are the propagation velocities of expansion-compression and shear waves in the medium and the outer layer of the shell.

Expressing displacement vectors in terms of Lame potentials

$$\mathbf{u}_k=grad\,\varphi_{1k}+rot\big(\varphi_{2k}\mathbf{e}_\eta\big)+rot\,rot\big(\varphi_{3k}\mathbf{e}_\eta\big),\quad k=1,\ 2,$$

we transform Eq. 4 to the form

$$\nabla^2\phi_{jk}=M_{jk}^2\frac{\partial^2\phi_{jk}}{\partial\eta^2},\ j=1,\ 2,\ 3,\quad k=1,\ 2. \tag{5}$$

Here M1k = Mpk, M2k = M3k = Msk.

Applying to (5) the Fourier transform in η, we find

$$\nabla_2^2\phi_{jk}^*-m_{jk}^2\xi^2\phi_{jk}^*=0,\quad j=1,2,3,\ k=1,2, \tag{6}$$

where ∇_2^2 is the two-dimensional Laplace operator,

$$m_{jk}^2=1-M_{jk}^2,\ m_{1k}\equiv m_{Pk},\ m_{2k}=m_{3k}\equiv m_{Sk},$$
$$\phi_{jk}^*(r,\theta,\xi)=\int_{-\infty}^{\infty}\phi_{jk}(r,\theta,\eta)e^{-i\xi\eta}d\eta.$$

Expressing the components of the stress-strain state (SSS) of the outer layer of the shell and its environment in terms of the Lame potentials and applying the Fourier transform in η, one can obtain expressions for the stress σ_{ijk}^* and displacement transformants

u_{ik}^{*} (k = 1, 2) in a cylindrical (i = r, θ, η, j = r, θ, η) coordinate system as a function of ϕ_{jk}^{*}.

Since the speed of the load is less than the speed of propagation of shear waves in the outer layer of the shell and the medium, then Msk < 1 (msk > 0) and solutions to Eq. 6 can be represented as:

- for environment

$$\phi_{j1}^{*} = \sum_{n=-\infty}^{\infty} a_{nj} K_n\left(k_{j1} r\right) e^{in\theta}, \tag{7}$$

- for the outer layer of the shell

$$\phi_{j2}^{*} = \sum_{n=-\infty}^{\infty} \left(a_{nj+3} K_n\left(k_{j2} r\right) + a_{nj+6} I_n\left(k_{j2} r\right)\right) e^{in\theta}. \tag{8}$$

Here In(kr), Kn(kr) are respectively modified Bessel functions and Macdonald functions, kj1 = |mj1ξ|, kj2 = |mj2ξ|; an1,…,an9 are unknown coefficients to be determined j = 1, 2, 3.

Applying to (3) the Fourier transform in η and expanding the displacement functions of the points of the middle surface of the shell and loads in Fourier series in θ, for the nth expansion term we obtain

$$\begin{aligned}
&\varepsilon_1^2 u_{0n\eta} + \nu_{02} n \xi_0 u_{0n\theta} - 2i\nu_0 \xi_0 u_{0nr} = G_0\left(P_{n\eta} - q_{n\eta}\right),\\
&\nu_{02} n \xi_0 u_{0n\eta} + \varepsilon_2^2 u_{0n\theta} - 2inu_{0nr} = G_0 (P_{n\theta} - q_{n\theta}),\\
&2i\nu_0 \xi_0 u_{0n\eta} + 2inu_{0n\theta} + \varepsilon_3^2 u_{0nr} = G_0 (P_{nr} - q_{nr}),
\end{aligned} \tag{9}$$

where $\varepsilon_1^2 = \alpha_0^2 - \varepsilon_0^2,\ \varepsilon_2^2 = \beta_0^2 - \varepsilon_0^2,\ \ \varepsilon_3^2 = \gamma_0^2 - \varepsilon_0^2,$

$$\xi_0 = \xi R_2,\ \ \alpha_0^2 = 2\xi_0^2 + \nu_{01} n^2,\ \ \beta_0^2 = \nu_{01}\xi_0^2 + 2n^2,$$

$$\gamma_0^2 = \chi^2\left(\xi_0^2 + n^2\right)^2 + 2,\ \varepsilon_0^2 = \nu_{01}\xi_0^2 M_{s0}^2,$$

$$\nu_{01} = 1 - \nu_0,\ \ \nu_{02} = 1 + \nu_0, M_{s0} = c/c_{s0},$$

$c_{s0} = \sqrt{\frac{\mu_0}{\rho_0}},\ \ \chi^2 = \frac{h_0^2}{6R_2^2},\ \ G_0 = -\frac{\nu_{01} R_2^2}{\mu_0 h_0},\ q_{nm} = (\sigma_{rm2}^{*})_n$ at r = R2; u0nm, Pnm are, respectively, the expansion coefficients $u_{0m}^{*}(\theta, \xi) = \int_{-\infty}^{\infty} u_{0m}(\theta, \eta) e^{-i\xi\eta} d\eta$ and $P_m^{*}(\theta, \xi) = \int_{-\infty}^{\infty} P_m(\theta, \eta) e^{-i\xi\eta} d\eta$ into Fourier series in the angular coordinate θ (m = η, θ, r).

Solving Eq. 9 with respect to u0nη, u0nθ, u0nr, we find

$$
\begin{aligned}
u_{0n\eta} &= \frac{G_0}{\delta_n}\sum_{j=1}^{3}\delta_{\eta j}\left(P_{nj} - q_{nj}\right),\\
u_{0n\theta} &= \frac{G_0}{\delta_n}\sum_{j=1}^{3}\delta_{\theta j}\left(P_{nj} - q_{nj}\right),\\
u_{0nr} &= \frac{G_0}{\delta_n}\sum_{j=1}^{3}\delta_{rj}\left(P_{nj} - q_{nj}\right).
\end{aligned}
\tag{10}
$$

.

Here δη = δ|η| = (ε1ε2ε3)2 – (ε1ξ1)2 – (ε2ξ2)2 – $(\varepsilon_3\xi_3)^2 + 2\xi_1\xi_2\xi_3$, $\delta_{\eta 1} = (\varepsilon_2\varepsilon_3)^2 - \xi_1{}^2$, $\delta_{\eta 2} = \xi_1\xi_2 - \xi_3\varepsilon_3{}^2$, $\delta_{\eta 3} = i(\varepsilon_3{}^2\xi_2 - \xi_1\xi_3)$, $\delta_{\theta 1} = \delta_{\eta 2}$, $\delta_{\theta 2} = (\varepsilon_1\varepsilon_3)^2 - \xi_2{}^2$, $\delta_{\theta 3} = i(\varepsilon_1{}^2\xi_1 - \xi_2\xi_3)$, $\delta_{r1} = -\delta_{\eta 3}$, $\delta_{r2} = -\delta_{\theta 3}$, $\delta_{r3} = (\varepsilon_1\varepsilon_2)^2 - \xi_3{}^2$, $\xi_1 = 2n$, $\xi_2 = 2\nu_0\xi_0$, $\xi_3 = \nu_{02}\xi_0 n$, for P_{nj} and q_{nj} index $j = 1$ corresponds to the index η, $j = 2 - \theta$, $j = 3 - r$.

To determine, for a fixed n, nine unknown coefficients an1,…,an9, we will use, depending on the condition of conjugation of the layers of the shell and its contact with the medium, the following boundary conditions, taking into account Eqs. 7, 8 and 10:

- with rigid conjugation of shell layers:
  - in case of sliding contact of the shell with the medium
    for r = R$_1$ $u^*_{r1} = u^*_{r2}$, $\sigma^*_{rr1} = \sigma^*_{rr2}$, $\sigma^*_{r\eta 1} = 0$, $\sigma^*_{r\theta 1} = 0$,
    $\sigma^*_{r\eta 2} = 0$, $\sigma^*_{r\theta 2} = 0$,
    for $r = R_2$ $u^*_{j2} = u^*_{0j}$, $j = r, \theta, \eta$,
  - in case of hard contact of the shell with the medium
    for r = R1 $u^*_{j1} = u^*_{j2}$, $\sigma^*_{rj1} = \sigma^*_{rj2}$,
    for $r = R_2$ $u^*_{j2} = u^*_{0j}, j = r,\ \theta,\ \eta$;
- in case of sliding conjugation of shell layers:
  - in case of sliding contact of the shell with the medium

    for r = R1 $u^*_{r1} = u^*_{r2}$, $\sigma^*_{rr1} = \sigma^*_{rr2}$, $\sigma^*_{r\eta 1} = 0$, $\sigma^*_{r\theta 1} = 0$,
    $\sigma^*_{r\eta 2} = 0$, $\sigma^*_{r\theta 2} = 0$,
    for r = R$_2$ $u^*_{r2} = u^*_{0r}$, $\sigma^*_{r\eta 2} = 0$, $\sigma^*_{r\theta 2} = 0$,
- in case of hard contact of the shell with the medium
    for r = R1 $u^*_{j1} = u^*_{j2}$, $\sigma^*_{rj1} = \sigma^*_{rj2}$,
    for r = R2 $u^*_{r2} = u^*_{0r}$, $\sigma^*_{r\eta 2} = 0$, $\sigma^*_{r\theta 2} = 0, j = r,\ \theta,\ \eta$.

Equating the coefficients of the Fourier-Bessel series at einθ, we obtain an infinite system (n = 0, ± 1, ± 2,…) of linear algebraic equations of block-diagonal form, the solution of which is found by a known method, if the corresponding for each n determinant $\Delta n(\xi, c)$ of the system is different from zero. After determining the coefficients an1,…,an9, applying the inverse Fourier transform, it is possible to calculate the SSS components of the array and the outer layer of the shell. In this case, any numerical method can be used to calculate the Fourier integrals, if the subsonic speed c of the load movement is less than its critical speeds, the values of which are determined in the study of the determinants $\Delta n(\xi, c)$.

By equating to zero symmetric with respect to n and ξ functions $\Delta n(\xi, c)$, dispersion curves in the (ξ, c) plane can be obtained numerically. For a fixed value of n, the coordinates $\xi(n)$, $c(n)$ of any point of the curve correspond to a free wave propagating along the axis of the shell. The shape of this wave depends on the number n and satisfies the corresponding homogeneous system of equations. Numerical studies of $\Delta n(\xi, c)$ carried out in (Alekseeva 1987) showed that, depending on the physical-mechanical and geometric parameters of the problem, for each n-mode there can exist a subsonic, corresponding to the minimum of the dispersion curve constructed in the (ξ, c) plane, critical speed $c = c(n)^*$, at which at two points $\pm\, \xi(n)^*$ $(\xi(n)^* > 0)$

$$\Delta_n\left(\pm\xi_{(n)*}, c_{(n)*}\right) = 0, \quad \partial\Delta_n\left(\pm\xi_{(n)*}, c_{(n)*}\right)/\partial\xi = 0.$$

In this case, there is no stationary solution of the problem for this mode. Moreover, the minimum critical speed occurs at $n = 0$. Therefore, if $0 < c < c(0)^*$, then $\Delta n(\xi, c) \neq 0$ for any ξ and n, and numerical methods can be used to calculate the integrals.

For $c(n)^* < c < \min csk$ $(k = 1, 2)$, for each n there are four singular points $\pm\, \xi(n)1$, $\pm\, \xi(n)2$ at which

$$\Delta_n\left(\pm\xi_{(n)l}, c_{(n)}\right) = 0, \quad \partial\Delta_n\left(\pm\xi_{(n)l}, c_{(n)}\right)/\partial\xi \neq 0, \quad l = 1,\ 2.$$

In these cases, a solution exists if the rank of the extended matrix is equal to the rank of the matrix of the system of equations for the given n-mode. For $c = c(n)^*$ the points $\xi(n)1$ and $\xi(n)2$ merge into one $\xi(n)^*$. There is no stationary solution to the problem in this case. At such speeds, resonant phenomena occur in the shell.

3.2 Numerical Experiment

Let us investigate the dynamic behavior of a steel ($\nu0 = 0.3$, $\mu0 = 8.08{\cdot}1010$Pa, $\rho0 = 7.8{\cdot}103$kg/m3) thin shell ($R2 = R = 1$m, $h0 / R = 0.02$) with a protective layer of thickness $hc = R1 - R2$ and without this layer in a rock mass with the following characteristics: $\nu1 = \nu = 0.25$, $\mu1 = \mu = 4.0{\cdot}109$Pa, $\rho1 = \rho = 2.6{\cdot}103$kg/m3; $cs1 = cs = 1240.35$ m/s. As an enclosing layer, let us consider:

- layer less rigid than rock mass – limestone layer ($\nu_2 = 0.25$, $\mu_2 = 2.8{\cdot}10^9$ Pa, $\rho_2 = 2.65{\cdot}10^3$ kg/m^3; $c_{s2} = 1027.9$ m/s);
- layer more rigid than rock mass – concrete layer ($\nu_2 = 0.2$, $\mu_2 = 1.21{\cdot}10^{10}$ Pa, $\rho_2 = 2.5{\cdot}10^3$ kg/m^3, $c_{s2} = 2200$ m/s).

We assume that the contact between the shell, the enclosing layer and the array is rigid.

On the lower half of the inner surface of the shell (90° ≤ θ ≤ 270°), a pressure load Pr moves at a constant speed c = 100 m/s, applied uniformly in the interval |η| ≤ 0,2R. Load intensity is P°.

The values of the critical load velocities c(|n|)* obtained as a result of the calculation for the shell without the enclosing layer and in the presence of such a layer of different thicknesses are presented in Tables 1, 2.

Table 1. Critical load rates for a shell without a protective layer.

h_0/R	$c_{(0)*}$, m/s	$c_{(1)*}$, m/s	$c_{(2)*}$, m/s	$c_{(3)*}$, m/s	$c_{(4)*}$, m/s	$c_{(5)*}$, m/s
0.02	1109	1110	1113	1127	1157	1177

Table 2. Critical load rates for a shell with a limestone enclosing layer.

h_c/R	$c_{(0)*}$, m/s	$c_{(1)*}$, m/s	$c_{(2)*}$, m/s	$c_{(3)*}$, m/s	$c_{(4)*}$, m/s	$c_{(5)*}$, m/s
0.1	1001	1002	1005	–	–	–
0.2	964	965	968	983	1012	–
0.3	955	956	959	974	1000	–
0.4	953	954	957	972	997	–
0.5	952	953	956	971	996	–
0.6	952	953	956	971	996	–
0.7	952	953	956	971	996	–
0.8	952	953	956	971	996	–
0.9	952	953	956	971	996	–
1.0	952	953	956	971	996	–

For a shell with a protective layer of concrete with the same ratios hc/R, as calculations have shown, the dispersion equations Δn(ξ, c) = 0 have no roots.

The results of calculations of the stress-strain state of the surface of the array or the enclosing layer in contact with the considered steel shell in the plane η = 0 are presented in Table 3. Changes in displacements and stresses when moving away in the radial direction from the lower point of the contour of the cross section η = 0 of the shell are given in Table 4.

Designations in the tables: u°r = ur μ/P° (m),
$\sigma^{\circ}_{rr} = \sigma_{rr}/P^{\circ}$, σ°θθ = σ°θθ/P°, $\sigma^{\circ}_{\eta\eta} = \sigma_{\eta\eta}/P^{\circ}$.

4 Discussion

From the analysis of the data in Tables 1 and 2, it follows that the creation of a layer around the shell, the rigidity of which is less than the rigidity of the medium, leads to

Table 3. SSS components of the contour of the contact surface $r = R = 1$m.

	θ, degrees						
	0	60	80	100	120	140	160	180
Shell without an enclosing layer								
u°_{r}	– 0.02	0.0	0.04	0.11	0.15	0.15	0.16	0.16
$\overset{\circ}{\sigma}_{rr}$	0.05	0.11	– 0.13	– 0.69	– 0.93	– 0.81	– 0.80	– 0.88
$\sigma^{\circ}_{\theta\theta}$	0.03	0.09	0.07	– 0.01	– 0.03	0.03	0.04	0.03
$\sigma^{\circ}_{\eta\eta}$	0.02	0.02	– 0.08	– 0.29	– 0.39	– 0.36	– 0.36	– 0.38
Shell with a limestone enclosing layer ($h_c/R = 0.1$)								
u°_{r}	– 0.02	0.0	0.04	0.12	0.16	0.17	0.17	0.18
$\overset{\circ}{\sigma}_{rr}$	0.06	0.11	– 0.13	– 0.69	– 0.93	– 0.81	– 0.80	– 0.87
$\sigma^{\circ}_{\theta\theta}$	0.03	0.08	0.05	– 0.06	– 0.09	– 0.04	– 0.02	– 0.04
$\sigma^{\circ}_{\eta\eta}$	0.02	0.02	– 0.07	– 0.26	– 0.35	– 0.32	– 0.32	– 0.35
Shell with a limestone enclosing layer ($h_c/R = 0.5$)								
u°_{r}	– 0.02	– 0.01	0.05	0.13	0.19	0.19	0.20	0.20
$\overset{\circ}{\sigma}_{rr}$	0.05	0.12	– 0.12	– 0.67	– 0.91	– 0.78	– 0.77	– 0.84
$\sigma^{\circ}_{\theta\theta}$	0.03	0.08	0.06	– 0.03	– 0.05	0.01	0.02	0.01
$\sigma^{\circ}_{\eta\eta}$	0.01	0.02	– 0.06	– 0.26	– 0.35	– 0.32	– 0.32	– 0.34
Shell with a concrete enclosing layer ($h_c/R = 0.1$)								
u°_{r}	– 0.02	0.0	0.03	0.08	0.12	0.13	0.13	0.13
$\overset{\circ}{\sigma}_{rr}$	0.03	0.13	– 0.13	– 0.72	– 0.98	– 0.85	– 0.82	– 0.88
$\sigma^{\circ}_{\theta\theta}$	0.11	0.09	0.20	0.36	0.47	0.49	0.46	0.44
$\sigma^{\circ}_{\eta\eta}$	0.02	– 0.03	– 0.17	– 0.42	– 0.56	– 0.56	– 0.58	– 0.61
Shell with a concrete enclosing layer ($h_c/R = 0.5$)								
u°_{r}	– 0.01	0.0	0.02	0.05	0.07	0.08	0.08	0.09
$\overset{\circ}{\sigma}_{rr}$	0.05	0.11	– 0.15	– 0.74	– 1.00	– 0.88	– 0.87	– 0.94
$\sigma^{\circ}_{\theta\theta}$	0.05	0.11	0.13	0.12	0.13	0.18	0.20	0.19
$\sigma^{\circ}_{\eta\eta}$	0.02	– 0.01	– 0.14	– 0.37	– 0.50	– 0.49	– 0.50	– 0.53

a decrease in critical load rates. At hc / R = 0.1, the critical speeds c(n)* are reduced by 10%. With an increase in the layer thickness hc, a further decrease in c(n)* occurs, which stops at hc / R = 0.5. As calculations have shown, the values of critical load rates at hc / R $\geq$ 0.5 in this case coincide with the values of critical load rates in a shell laid in a limestone massif.

To increase the critical load speeds, a more rigid enclosing layer should be used, for example, using a concrete layer as such a layer.

From the analysis of the results of calculations (Tables 3 and 4) it follows that when the shell is enclosed with a limestone layer less rigid in relation to the massif, the largest

Table 4. Changes in displacements and stresses with distance from the shell.

		r/R						
	1	1.2	1.3	1.4	1.5	2.0	2.5	3.0
Shell without an enclosing layer								
u°_{r}	0.16	0.11	0.09	0.07	0.06	0.03	0.02	0.01
$\overset{\circ}{\sigma}_{rr}$	– 0.88	– 0.54	– 0.39	– 0.29	– 0.22	– 0.08	– 0.04	– 0.02
$\sigma^{\circ}_{\theta\theta}$	0.03	0.06	0.05	0.04	0.03	0.01	0.01	0.0
$\sigma^{\circ}_{\eta\eta}$	– 0.38	0.0	0.03	0.03	0.03	0.01	0.01	0.0
Shell with limestone enclosing layer (h_c/R = 0,5)								
u°_{r}	0.20	0.13	0.10	0.08	0.06	0.03	0.02	0.01
$\overset{\circ}{\sigma}_{rr}$	– 0.84	– 0.52	– 0.39	– 0.30	– 0.23	– 0.08	– 0.04	– 0.02
$\sigma^{\circ}_{\theta\theta}$	0.01	0.04	0.03	0.01	0.0			
0.03	0.01	0.01	0.0					
$\sigma^{\circ}_{\eta\eta}$	– 0.34	0.0	0.02	0.01	– 0.01			
0.02	0.01	0.01	0.0					
Shell with concrete enclosing layer (h_c/R = 0,5)								
u°_{r}	0.09	0.07	0.06	0.05	0.05	0.03	0.02	0.01
$\overset{\circ}{\sigma}_{rr}$	– 0.94	– 0.56	– 0.38	– 0.26	– 0.18	– 0.07	– 0.03	– 0.02
$\sigma^{\circ}_{\theta\theta}$	0.19	0.21	0.20	0.19	0.20			
0.03	0.01	0.01	0.0					
$\sigma^{\circ}_{\eta\eta}$	– 0.53	– 0.01	0.07	0.13	0.21			
0.04	0.01	0.01	0.0					

radial displacement ur of the cavity surface increases, and the values of the largest normal stresses |σrr|, |σηη| and |σθθ| decline. When using concrete, which is more rigid than the enclosing layer, the opposite effect occurs.

With distance from the shell, the displacements and stresses mainly decay, and at $r/R \geq 3.0$ they become almost insignificant. When crossing the boundary between the enclosing layer and the surrounding massif, the values of the stresses σηη and σθθ change abruptly.

5 Conclusions

Summarizing the research results presented in the paper, it can be noted that:

- by changing the parameters of the layer enclosing the tunnel lining from the array, it is possible to increase or decrease the value of the critical speed of the load, as well as change the stress-strain state of the array;
- the greatest thickness of the dynamically active layer around the shell is about two of its radii.

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